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Geometric Algebra as a Unifying Language

Abstract

Geometric Algebra (GA) is a powerful unifying language applicable to many branches of mathematics, physics and engineering. One of the main aims of the talk will be to demonstrate how the same mathematical tools in GA can be used for both engineering purposes and advanced physics topics, thus making the latter more approachable and understandable for a greater number of people. A selection of topics from three main areas will be discussed, with the emphasis on each partially determined by the topics which arise during the conference itself. These are:

(a) Progress towards a scale-invariant theory of gravity. A sketch will be given of how using GA, gravity can be rewritten as a gauge theory in flat space, making it much more similar to our other theories of fundamental forces. The gauge symmetries involved are then extended to include scale-invariance and some physical aspects of the resulting theory are discussed.

(b) In common with some other talks this week, we discuss the use of GA in Electromagnetism (EM). A topic which has only been explored in a limited way before within a GA approach, is the aspect of the apparent 'time asymmetry' in electrodynamics, in which we see only retarded solutions of the radiation equations, not advanced solutions. This currently has no explanation in terms of EM theory itself, and is potentially linked with boundary conditions in the future expansion of the universe, for example in the Wheeler-Feynman 'absorber' theory of radiation. We demonstrate, possibly for the first time, an interesting intrinsic asymmetry between incoming and outgoing radiation, which may be relevant to this question.

(c) A '1d-up' approach to conformal geometric algebra (CGA). The CGA has been finding widespread use, for example in computer graphics and other areas where the ability to carry out high level geometric manipulation of objects is needed. In the usual formation, two extra dimensions are needed in which to express the CGA, involving the introduction of two new null vectors. In the '1-d up' approach, we introduce just one extra vector, with unit square. This is achievable by working not in a Euclidean space, but a space of constant curvature. We show that in an engineering context, this can provide new error models for matching of groups of points, lines or planes, under rigid motions, and how these are fully covariant under translations and rotations. In a physics context, we show how cosmology and electromagnetism take on a very different character using these new spaces, and how a particular type of de Sitter space is singled out as one which may be the preferred arena

for physics. This also ties back to topic (a), where the new scale invariant gravity theory discussed there, is shown to possess not just scale-invariance, but (just like electromagnetism) invariance under the full conformal group.