A systematic study of multi-layer canard cycles

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Given a slow-fast system containing several layers, we study the canard cycles transiting through these layers. All the canard cycles we consider are topological circles and in between the layers we restrict to generic Hopf breaking mechanisms and generic jump breaking mechanisms. There is a difference in between transiting a terminal layer or a transiting a dodging layer, but more important reveals to be the precise way in which succesive layers are connected to each other. At such a breaking mechanism we have to compare a transition map coming from an attracting sequence on one side to a transition map coming (in negative time) from a repelling sequence on the other side. The comparison can be side-preserving or side-reversing and this makes a major difference in the calculations, affecting strongly the results on the number and the possible bifurcations of the limit cycles. We provide an easy criterion to see whether at a breaking mechanism the comparison is side-preserving or side-reversing and we introduce a "connection diagram" associated to a multi-layer canard cycle. We show that all possible abstract connection diagrams can be realized by a multi-layer canard cycle of a planar polynomial system. We also precisely state the conditions under which a connection diagram can be realized by a multi-layer canard cycle of a (polynomial) Liénard equation. The connection diagram presents in a clear way all the available information that is necessary to get the system of equations describing the bifurcating limit cycles. We provide some general results concerning these equations, leading already to precise consequences on the cyclicity and

the occurring bifurcations for all possible types of 2-layer canard cycles.