

Optical Billiards

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Lignières and Georget, in 2009, presented an asymptotic analysis of high-frequency acoustic modes in rapidly rotating stars, based on acoustic ray dynamics. They remark that, as the star rotates, its boundary is a deformation of a sphere on the equatorial plan, i.e. it is a 4-periodic symmetric ovoid. Then they show that, using the symmetries of rotation, the acoustic rays can be described in two ways: either as trajectories of a particle under a classical 2-degree of freedom Hamiltonian depending on the frequency of rotation of the star, with a potential tending to infinity when approaching the boundary of the star; or as trajectories of optical rays in an isotropic 2-dimensional medium, with medium index depending on the distance to the center of the star.

This analysis produces a new kind of billiard problem, called optical billiard, where an optical ray travels along the geodesics of a Riemannian metric associated to the medium index, which depends only on the distance to the origin, performing elastic reflections at the impacts with the boundary, where elastic means angle of incidence equal angle of reflection when measured by the internal product induced by the Riemannian metric.

The motion is then completely determined by the point of reflection at the boundary and the direction of movement immediately after each reflection. So, a parameter θ , which locates the point of reflection, and the angle φ between the direction of motion and the tangent to the boundary at the reflection point, may be used to describe the system.

The billiard model defines then a bidimensional map, which associates to each impact coordinate and direction of motion (θ_0, φ_0) the next impact and direction (θ_1, φ_1) . If the boundary is a C^k closed geodesically convex curve then the billiard map is a C^{k-1} Twist reversible and conservative diffeomorphism, having some of the generic properties as the usual convex billiard in the plane.

This is a joint work with Cássio H. V. Morais (UFES), Rafael da Costa Pereira (UFMG) , Mário J. Dias Carneiro (UFMG) and Sylvie Oliffson Kamphorst (UFMG).
