Catastrophe theory for modern mathematical modelling

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Many of us are familiar with the basic singularities or bifurcations: folds or 'saddle-nodes', cusps, Hopf bifurcations, and so on. And the list goes on: swallowtails, butterflies, wigwams, stars,... But did you know that there are low order bifurcations of vector fields that have never (seemingly) been classified? I'll show you a singularity where 4 equilibria of a vector field collide that seems to have eluded traditional classifications. And then, given a complicated physical or biological model, how do you discover if and where any given singularity occurs in its system of equations? Did you know that finding a 4-parameter bifurcation in 4 dimensions could take at least 95 million calculations, to find a 5-parameter bifurcation in 5 dimensions could take 3×10^{30} calculations! It turns out that while traditional singularity theory is great at classifying all the bifurcations that are possible (well, not 'all' as we've discovered), it is terrible if you want to find where a given singularity or bifurcation happens. But it turns out that there is an alternative: instead of the singularity or bifurcation itself, you look for its "underlying catastrophe". The underlying catastrophe doesn't care whether you're studying a map or a vector field or even a PDE, it doesn't care how many dimensions your system has ... to locate an r-parameter singularity or bifurcation will take precisely r calculations, and no more. Since I proposed this preposterous idea 3 years ago, and applied it to find a butterfly singularity in a biological reaction diffusion related to plant growth, I have been working to understand how it connects to the deep singularity theory set down by Thom, Arnold, Mather, with some wonderful ideas and almost 'lost' bifurcations you may not have seen from such characters as Dumortier and Sotomayor. I'll introduce you to the idea of underlying catastrophes, show you some singularities and bifurcations you (probably) have never seen, and show you new ways to investigate them in contexts (like vector fields and PDEs) where they couldn't be investigated before, opening the door to studying their wider role in applications to pattern formation and phase transitions.