Hopf bifurcation of limit cycles surrounding a node

Jaume Llibre
jaume.llibre@uab.cat
Universitat Autònoma de Barcelona, Spain

We consider the planar polynomial differential systems in $\mathbb{R}^2$

$$\begin{aligned}\dot{x} &= \mu x + P(x,y,\mu),\\ \dot{y} &= \mu y + Q(x,y,\mu),\end{aligned}$$

where the polynomials P and Q have neither constant terms nor linear terms, satisfying that when the parameter $|\mu| \neq 0$ and small, the origin is a node, and for $\mu = 0$ the origin is either a node or a focus. The main theorem characterizes the Hopf bifurcation from the equilibrium point at the origin of this system. We illustrate this result with several examples. As a consequence, we can give a lower bound for the number of small limit cycles surrounding a node depending only on the degree of the polynomial differential system. In summary, we extend the Hopf bifurcation that usually studies the bifurcation of limit cycles from a focus to the bifurcation of limit cycles from a node.

In this talk the node at the origin is an star node, but the results that I will present for the star node also work for the standard node $\dot{x} = \mu x$, $\dot{y} = \lambda y$ with $\mu\lambda > 0$, or for the non-diagonalizable node $\dot{x} = \mu x + y$, $\dot{y} = \mu y$.