



V Workshop in Stochastic Analysis and Applications

IMECC - Unicamp

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Applied rough analysis: from neural differential equations to signature kernels.

Abstract

A core achievement of Lyons' theory of rough paths is to provide a robust solution framework to controlled differential equations (CDEs) driven by irregular signals. The crux is to enhance the driving path with iterated integrals, up to a certain order, to form a rough path. The signature, a core object in the theory, is the solution to a particular CDE whose coordinates emerge as a universal feature set, or "basis", to describe functions on unparameterized paths, and serve as a foundation for functional approximation. In the first part of this minicourse, we will set the scene by introducing the signature transform and its main analytic and algebraic properties.

In deep learning, neural ordinary differential equation (Neural ODE) models have recently emerged, offering a continuous-time approach for modelling the latent state of residual neural networks. Neural controlled differential equations (Neural CDEs) are CDEs with vector fields parameterised by neural networks. Neural CDEs can be thought of as continuous-time analogues of recurrent neural networks and provide a memory-efficient way to model functions of incoming data streams. In the second part of this minicourse, we will provide an overview of the fundamental constituents of rough path theory, placing emphasis on how they are used in practice for the training of Neural CDEs.

Signature kernels are defined as certain inner products of signatures and

provide a valid alternative to signatures for learning with high dimensional time series. In the third and final part of this minicourse, we will provide an overview of the theory of signature kernels, exhibiting several constructions of signature kernels, including infinite-width limits of randomised CDEs.